

Proportional Representation, Majoritarian Legislatures & Coalitional Voting

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Abstract

Voters in elections under plurality rule face relatively straightforward incentives when it comes to voting. Voters in PR systems face more complex incentives as electoral outcomes don't translate as directly into policy outcomes as in plurality rule elections. A common approach is to assume electoral outcomes translate into policy as a vote-weighted average of all party platforms. Most of the world's legislatures are majoritarian institutions and elections in PR systems are generally followed by a process of coalition formation. I demonstrate that existing results are not robust to the introduction of minimal forms of majoritarianism. Voters' incentive to engage in strategic voting are shown to depend on considerations about the coalitions that may form after the election. In line with the empirical findings in the literature, the voters' equilibrium strategies are shaped by policy balancing and the post-electoral coalition bargaining situation, including considerations about who will be appointed the formateur.

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1 Introduction

Voters' ability to influence policy is central to the idea of representative democracy (Powell 2000). The theoretical literature on how voters' preferences, or choices, are translated into policy has provided important insights. It has explored how electoral systems influence the menu of choices facing voters – including the number of alternatives available to them as well as the policies advocated by those parties or candidates – and, finally, how voters' choices from the set of those alternatives affect policy outcomes. The initial efforts focused, perhaps disproportionately, on electoral competition in single member districts under plurality rule. In recent years scholars have to an increasing degree extended their effort to the study of a wider variety of the world's electoral systems including majority run-off elections (Callander 2005), the single transferable vote (Jesse 1999), mixed-member systems (Bawn & Thies 2003), and, last but not least, proportional representation systems (Cox & Shugart 1996). This paper is concerned with the last of these systems.

The task of modeling voter behavior is greatly simplified when electoral outcomes can be assumed to translate directly into policy outcomes. In elections under plurality rule, for example, it is often reasonable to assume that the plurality winner simply implements his preferred (or promised) policy outcome. The strategic calculations under proportional representation are widely recognized to be more complicated because the number of ways in which a voter can be pivotal increases with the number of candidates elected within a given district. Perhaps less acknowledged source of strategic motivations is the fact that voters may care less about the parties' vote shares than how they influence policy outcomes. Thus, how vote shares are aggregated into policy outcome is a crucial issue that has received considerable attention in the literature on coalition formation.¹ Importantly, it is not necessarily the case that the relationship between vote share and influence on policy outcome is monotonic. To act strategically, the voter must consider how his vote feeds through the coalition formation process to influence the policy outcome, which may mean that voting for one's preferred party is not necessarily the optimal strategy.

The common wisdom has been that strategic voting is largely absent in proportional representation systems – a view that is in large part predicated on the notion that it is difficult for voters to figure out how to maximize their likelihood of casting a pivotal

¹The literature on coalition formation has generally not considered how voters' strategies are influenced by the policy determining processes that are initiated following an election. For an important exceptions see Austen-Smith & Banks (1988) and Austen-Smith (2000).

vote. Recent research has gone some way towards overturning that view voters don't vote strategically in proportional representation elections. Aldrich et al. (2004) and Blais et al. (2006) find that a substantial proportion of voters don't vote for their most preferred party and, more importantly, that preferences over coalitions and potential formateurs influence vote choice. Kedar (2005) finds that voters engage in policy balancing by voting for parties that take more extreme positions than their most preferred party. Bargsted & Kedar (2009) also show that expectations about who will form coalition influence vote choice.

The presence of strategic voting in proportional representation systems raises important questions about the perceived advantages and disadvantages of proportional representation. First, proportional representation is often regarded fair in normative terms as each group's representation in the legislature is proportional to its size. If strategic voting is prevalent this claim loses much of its force. Secondly, and more importantly for our present purposes, strategic voting has the potential to influence a number of political outcomes that we care about. In voting strategically voters are likely to desert certain parties in favor of certain other parties (e.g., those who are likely to join a coalition), thus, influencing the size and the shape of the party system. At the same time the voters' choices influence which coalitions form and what policies are implemented.

Understanding voter behavior in proportional representation systems is, therefore, fundamental to understanding how the preferences of the electorate are translated into policy outcomes and in evaluating the performance of different electoral systems. The (formal) theoretical literature on voter behavior in proportional representation systems is rather limited and one might say that theoretical developments haven't fully kept up with the empirical work in the area. In the next section I briefly review the formal literature and argue that it makes strong assumptions about the policy making process that don't chime well with what is known about parliamentary systems. In this paper, I begin by showing that the results of the standard model of voting in proportional representations systems are sensitive to relatively minor changes in its assumptions about the policy-making process. I then consider a simple model of coalition formation that highlights voters' behavior is influenced by coalition considerations, giving rise to both what the empirical literature has termed coalitional voting and policy balancing.

2 Models of Voting in Proportional Representation Systems

Much of the literature on proportional representation assumes that the policy, p , implemented following an election equals the vote-weighted (or seat-weighted) average of the parties' ideal policies, or:

$$p = \sum_{k \in K} v_k x^k \tag{1}$$

where K is the set of competing parties, v_k is the vote share received by party k , and x^k is party k 's ideal policy or policy platform.² This assumption has been used to study various aspects of electoral competition under proportional representation.³ One of the central questions here has been what kinds of policy platforms parties adopt when competing proportional representation system. The general flavor of the results is given by a simple position-taking model of two party competition in which voters vote sincerely; the candidates adopt extreme positions (Ortuño-Ortín 1997). Two party competition is not typical of proportional representation systems but the finding is replicated in the more realistic case where the formation of parties is endogenous and voters behave strategically (Gerber & Ortuño-Ortín 1998, Gomberg, Marhuenda & Ortuño-Ortín 2004). In the context of the citizen-candidate model candidates' positions are similarly found to depart significantly from the median voter's preferred policy (Hjortlund & Hamlin 2000). These results are driven by the fact that voters vote strategically for extremist parties. If the parties' platforms are exogenously chosen, only the most extreme parties on each end of the political spectrum receive votes (De Sinopoli & Iannantuoni 2007).⁴ The results of these models are useful in as much they comport with the perceived wisdom that proportional representation system exhibit greater ideological diversity than majoritarian systems. However, there is no evidence suggesting that extremist parties will be dominant in elections under proportional representation as these results would suggest.

²I restrict my discussion here to models that assume vote-weighted policy outcomes. There are a handful of models, e.g., Austen-Smith & Banks (1988), Austen-Smith (2000), and Persson & Tabellini (2000), that pay greater attention to the policy making process. In short, scholars would be well advised to focus their efforts on extending these models.

³Note that this assumption is quite different from assuming that policy outcomes equal the weighted average of the policy positions of the members of the governing coalition. While the latter assumption is quite common in the literature, its implication for voter behavior have not been considered in much detail.

⁴The vote-weighted assumption appears in a slightly different form in the redistributive models such as Myerson (1993), Crutzen & Sahuguet (2009) and Sahuguet & Persico (2006) where the likelihood of the candidates' proposed distributions is implement with a probability equal to their vote share or, in Myerson's case, each party allocates a share of the budget equal to its vote share.

The assumption of vote-weighted policy outcomes also appears widely in articles considering the consequences of electoral institutions for a variety of other substantive topics. The assumption appears, e.g., in models showing that conflict or civil war is less likely in proportional representation systems (Reynal-Querol 2002, Reynal-Querol 2005, Esteban & Ray 2008), that the provision of public goods is greater (Lizzeri & Persico 2001), that budget deficits are not affected (Lizzeri 1999), that corrupt politicians are less likely to be elected (Myerson 1993), that interest groups can gain influence by threatening contributions to other parties (Chamon & Kaplan N.d.), and that parties that are expected to win a majority are more likely to cater to special interest groups (Grossman & Helpman 1996). The assumption also appears in experimental and computational work. Schram & Sonnemans (1996), e.g., study the effects of electoral institutions on turnout in an experimental setting and Kollman, Miller & Page (1997) consider sorting in a Tiebout model with computational methods.

There are a number of reasons why one might object to the assumption of vote-weighted policy outcomes. Most importantly, it implicitly assumes that *all* the parties elected to the legislature influence the policy outcome. This essentially amounts to unanimity rule in the sense that no party can be shut out of the decision making process.⁵ Legislatures, by and large, are, however, majoritarian institutions and whoever commands majority of the seats in the legislature usually gets his or her way.⁶ A governing coalition is formed following elections in parliamentary systems, and that coalition holds the reins of power until the end of the electoral term or until it loses the legislature's confidence. Research on coalition formation show that most such coalitions are majority coalitions – sometimes they are supermajoritarian, but “unanimity” coalitions are extremely rare.⁷ Minority coalitions are also fairly common but research into how such coalitions build support for its legislative agenda is scarce. In some places, e.g., Denmark, such coalitions advance its

⁵To take a simple example suppose a moderate party receives 51% of the seats while another party receives 49%. It might be reasonable to assume a single party ‘coalition’ forms and implements its ideal policy – compromise may be unlikely if the parties are far apart ideologically as the larger party has a parliamentary majority. The assumption of vote-weighted policy outcomes implies, however, that the resulting policy would be roughly at the midpoint between the two parties – the policy can thus be pulled far away from the median voter (and the major party) if the smaller party is very extreme.

⁶I am not aware of any legislatures that do not operate under majority rule. Some legislatures required a supermajority to pass certain types of bill, e.g., in Finland a 2/3 majority is required for the budget to pass (Nousiainen 2001). Similarly, in Hungary a 2/3 majority is required to pass ‘major’ legislation (Gallagher, Laver & Mair 1995).

⁷Most, if not all, of these cases are so-called wartime coalitions formed in circumstances of international crises.

agenda by building support for its legislative bills on a piecemeal basis, i.e., by forming coalitions around individual bills that may consist of different parties (Damgaard 2001). There is, however, no evidence suggesting that such strategies necessarily allow *all* parties equal (or proportional) influence in the legislative bargaining and *a priori* that possibility appears somewhat implausible. For most polities, the assumption of vote-weighted policy outcomes therefore appears to be a poor description of the policy making process and, moreover, it fails to capture important aspects of how policy is made.

The assumption of vote-weighted policy outcomes has sometimes been justified as “being a reasonable description” of policy making.⁸ Although most governing coalitions are majoritarian it does not preclude the possibility that legislatures play an important and a substantive role in policy making. Proportional representation systems are most common in parliamentary systems. The consensus on parliamentary systems is that parliaments’ power derives, first and foremost, from the fact that governing coalitions must retain its confidence.⁹ When it comes to influencing legislation, the ability of parliament – in large part thanks to the high levels of party discipline in parliamentary systems – can best be described as marginal. While there are certainly differences cross-nationally in the institutional capabilities of parliaments to influence policy (Strøm 1984), the evidence suggesting that these institutional factors have a substantial impact is limited.¹⁰ Moreover, these parliamentary institutions may be used strategically by the governing coalition to monitor the legislative actions of its coalitions partners rather than offering the opposition a piece of the legislative pie (Vanberg & Martin 2004).¹¹ Finally, equal access to legislative

⁸Hjortlund & Hamlin (2000, p. 206), e.g., argue that “[t]his might be interpreted either as an explicit voting rule that determines policies directly as a result of the pattern of votes received by the various candidates, or as the reduced form of a more complex political process in which candidates are first elected to an assembly which then chooses policies. We prefer the second interpretation, where our analysis is not tied to specific voting procedures at any stage of the political process, but seeks to capture the overall relationship between popular voting for candidates and final policy outcomes.” Ortuño-Ortín (1997, p. 429) admits that the assumption is unrealistic but that it captures but justify it by arguing that “it is hard to believe that it makes no difference to win with 51% or to win with 90% of the votes.” Gomberg, Marhuenda & Ortuño-Ortín (2004, p. 377) “believe that this is a realistic assumption, which captures the way many democratic societies adopt policies.”

⁹See, however, Huber (1992) and Diermeier & Feddersen (1998) who show how the confidence votes enhances party discipline.

¹⁰See Powell (2000) for a recent assessment of the ability of opposition parties to influence policy-making. In general, however, there is very little concrete evidence concerning oppositional influence as the discipline appears to have shown limited concern for studying, or tracking, actual policy outcomes.

¹¹Furthermore, the fact that governing coalitions use the legislature as a venue to keep tabs on one another suggest that they are not particularly concerned about giving the legislature an opportunity to scrutinize their bills.

influence, independent of ideological positions, appears highly unlikely. In France, for example, the center right parties have repeatedly refused to cooperate with the National Front. How often parties are rolled in the legislature has been shown to depend on their ideological position.¹² Overall, at least as far as parliamentary systems are concerned, there remain great doubts that the assumption of vote-weighted policy outcomes can be justified on the basis of “realism”.

Another possible justification suggests that the assumption describes how voters expect their votes to influence the policy outcome, even if the assumption does not accurately describe the nuts and bolts of policy making. That is, voting for a particular party may increase the likelihood of the party being included in the government coalition – perhaps because it increases the party’s likelihood of being picked as a formateur or its opportunity of government participation increases in other ways. However, if one is to claim that voters vote as if the parties have proportional influence on the final outcome, then that theory is an urgent need of micro-foundations as empirically it is not the case that the final outcome of parliamentary bargaining is given by equation (1). If, on the other hand, voters have a more nuanced view of the policy-making process it would be reflected in their choices, e.g., if the governing coalition monopolizes policy-making it would rarely make sense for a voter to cast a vote for an extremist party with limited chances of entering a coalition. Hence, to accurately describe the incentives for strategic behavior that voters face it is essential that our model captures the main characteristics of the policy-making process.

The model considered here demonstrates the importance of making appropriate assumptions about the policy-making process. The directional theory of voting (Rabinowitz & Macdonald 1989) is often considered the proximity model’s main contender. It posits that voters seek to influence the direction of policy change and to do so they may vote for parties that don’t necessarily represent their preferences the best. Rabinowitz & Macdonald’s (1989) theory doesn’t provide an explanation for why voters don’t carry the reasoning to its logical conclusion and vote for extremist parties. Rather, voters are assumed to limit their choices to some ‘region of acceptability’. As we shall see, even though it is assumed here that voters seek to minimize their distance from the policy outcome, the voters’ incentives don’t always demand that they vote for the most proximate party. I show that their behavior resembles what would be expected if voters behaved as prescribed by the di-

¹²See, e.g., Cox, Masuyama & McCubbins (2000) and Amorim Neto, Cox & McCubbins (2003). A party is “rolled” when a legislation or an amendment is adopted against the will of the party’s members.

rectional theory. Moreover, when coalitions form following an elections, some voters may not vote for the most proximate party but their incentives to vote for extremist parties are nonetheless tempered by the desire to influence the policy outcome. In other words, something akin to ‘regions of acceptability’ arises endogenously in the model. Thus, it is not clear that the evidence that has been considered to favor the directional theory is inconsistent with the proximity model when the voters’ incentives are modeled correctly.

The assumption of vote-weighted policy outcomes has been used widely in the formal literature as pointed out above even though there is no empirical evidence to suggest that voters desert moderate candidates in favor of extremists. Models employing this assumption have also, to an increasing degree, been used to motivate empirical work. Given how implausible the assumption is theoretically, it is important to consider how robust it is to modifications that incorporate some degree of parliamentary majoritarianism. I begin by considering a slightly modification of the assumption of vote-weighted policy outcomes that allows majority winners to decide on policy unilaterally. Although it is difficult to envision a smaller role for majoritarian policy making than this, it has a significant impact on the equilibrium of the game. The voters’ incentive to vote for radical parties is reduced and equilibrium policy outcomes are closer to the median voter’s preferred outcome.

In order to examine more closely how voter behavior responds to the majoritarian nature that characterizes most legislatures I then consider a simple coalition formation game where the parties’ vote shares and policy platforms only influence the policy outcome if they are members of the government coalition. It bears noting that my primary focus here is not on coalition bargaining but rather demonstrating how the introduction of a coalition formation process has clear implications for the voters’ equilibrium strategies that include both incentives to vote for coalitions as well as against coalitions. The two types of incentives generated by the stylized coalition formation process clearly resemble the types of strategic behavior that has been noted in the empirical literature. The former stems from the voters’ ability to influence the policy outcome associated with a given coalition while the latter incentive centers on influencing how attractive the formateur finds different coalition partner. The incentive to vote for extremist parties is further reduced in the simple coalition formation model and policy outcomes tend to be located close to the median voter’s preferred policy.

3 Proportional Representation & Majoritarian Legislatures

I begin by considering a model of proportional representation with a fixed number of parties $r > 2$, each of which is characterized by its (exogenously given) policy platform $p^k \in \mathbb{X} \subset \mathbb{R}$, and a distribution of strategic voters whose preference profile is single-peaked on \mathbb{X} . Assuming vote weighted policy outcomes, De Sinopoli & Iannantuoni (2007) show that the game (with a discrete number of voters) has essentially a unique Nash equilibrium with the characteristics that there are only two vote-receiving parties and, in addition, the two parties are those with the most extreme policy positions.¹³ The resulting policy outcome is centrist as it equals the vote-weighted average of the parties' ideal policies. The logic behind the result is similar in spirit to the results obtained by Alesina & Rosenthal (1996), in the context of divided government, and Austen-Smith (1984), in the context of plurality elections in multiple constituencies. That is, by voting for extreme parties the voters are able to inch the policy outcome towards their ideal policy with the end result that a 'moderate' policy outcome obtains in equilibrium. Thus, the vote-weighted model of proportional representation generates two predictions that are perplexing when put in empirical context. First, Duverger's *hypothesis* (Riker 1982) is incorrect and proportional representation leads to a two party system. Second, politics in proportional representation systems are dominated by extremist parties.

Given that these predictions don't resonate with real-world electoral outcomes, it is important to examine what gives rise to the predictions. The results clearly rely on the assumption that the legislature is not a majoritarian institution. If only two parties receive votes in equilibrium, one party must hold a majority of the seats in the legislature. In most legislatures, the larger party would be expected to form a single party cabinet and it is not obvious why a majority party would choose to make compromises to the minority party. Yet, in the vote-weighted model, the voters' strategies are predicated on such a compromise, which provide them with an incentive to vote for extremist parties. If voters expected a single party cabinet to form, it appears likely that they would be more reluctant to vote for extremist parties.

I begin by reconsidering the basic vote-weighted model with the (minimal) modification that the vote-weighted mapping from electoral outcomes to policy outcomes is conditional

¹³De Sinopoli and Iannantuoni also offer a result with a continuum of voters that fully characterizes the equilibrium by a unique policy outcome.

on no party receiving a majority of the vote. If a single party wins a majority, it alone determines the policy and, consequently, implements its policy platform.

The Model

The policy space is a closed interval, $\mathbb{X} \subset \mathbb{R}$. There is a set of r parties, $K = \{1, 2, \dots, r\}$. Each party, $k \in K$, has a fixed policy platform, $p^k \in \mathbb{X}$, and the parties are labelled so that $p^k < p^{k+1}$. There is a continuum of voters whose preferences are single-peaked over \mathbb{X} . Each voter's preferences are characterized by his ideal point $x^i \in \mathbb{X}$. Let $u_i(x)$ be voter i 's concave utility function. The distribution of the voters' ideal points is described by the density function $f(x)$. Let $F(x)$ be the cdf of $f(x)$ and assume $F(x)$ is continuous and symmetric about the median of the distribution. Each voter can cast a vote for a party in K . The voter's set of actions is A_i where $k \in A_i$, a vote for party k , is a vector of length r containing zeroes except for the k^{th} component. Let $A = \times_{i \in N} A_i$ and $A_{-i} = \times_{i \in N \setminus \{i\}} A_i$. The voter's strategy is a mapping $s_i : A_{-i} \times \mathbb{X} \rightarrow A_i$. Abusing notation slightly let $s_i = k$ be a shorthand for $s_i = \{0, \dots, 0, 1, 0, \dots, 0\}$ where 1 is the k^{th} component of the vector. For a given strategy profile s , let $v(s)$ be the vector of the parties' vote shares and $v_k(s)$ denote the share of the vote received by party k . When no party wins a majority of the vote, the policy outcome, $p(v(s))$, is a convex combination of the parties' policy platforms, where the vote share determines the weight of each party. If some party wins a majority of the vote it implements its policy platform. The policy outcomes equals:

$$p(s) = \begin{cases} p^k & \text{if } v_k(s) > \frac{1}{2} \text{ for some } k \in K \\ \sum_{k \in K} p^k v_k(s) & \text{else} \end{cases} \quad (2)$$

Let $L(p) = \{i \in N | x^i < p\}$ and $R(p) = \{i \in N | x^i > p\}$ be the sets of voters that prefer, respectively, a more 'left-leaning' or a more 'right-leaning' policy than the policy p . The median voter, m , is defined by $F(x^m) = \frac{1}{2}$. Let $k^m = \arg \max_{k \in K} u_m(p^k)$ be the median voter's most preferred party – for simplicity it is assumed that the median voter is not indifferent between his two most preferred parties. Henceforth party k^m will be referred to as the *median preferred party*.

Consider the Nash equilibria of the game. With a continuum of voters each voter's

action does not influence the outcome.¹⁴ I assume, however, that the voters behave as if their vote could influence the outcome of the election. It is immediately obvious, much like in multicandidate elections under plurality rule, that the game has multiple Nash equilibria. Eliminating weakly dominated strategies has limited purchase in the game because a vote for each party can be a best response for all but the most extreme voters.

Proposition 1 *Almost any strategy profile such that $v_k(s) > \frac{1}{2}$ for some $k \in K$ is a Nash equilibrium.*

The proposition follows directly from the fact that the elimination of weakly dominated strategies eliminates hardly any strategies for each voters. It is only possible to eliminate strategies for voters whose ideal points are more extreme than those of the two most extreme parties and in those cases the only strategies eliminated are for the party least preferred by these voters. The ‘almost’ in the statement of the proposition applies to situations in which a large share of voters is more extreme than all the parties. The proof of the proposition is straightforward and is omitted.

The set of equilibria thus expands rather dramatically when a majoritarian element – in its simplest possible form – is added to the model. Proposition 1 is rather unsatisfying because it neither gives us a clear prediction about which party wins the elections nor a prediction about the number of (vote receiving) parties. In addition, many of the possible equilibria do not seem very ‘reasonable’, i.e., extremist parties have their policy platforms implemented even when more moderate parties, whose election would leave a great majority of the voters better off, are present. The reason such equilibria exist is that voters are implicitly assumed to be unable to coordinate their actions in any shape or form. In reality voters are likely to rely on a variety of ways to coordinate, e.g., cues from opinion leaders, political polls, etc. One way to move forward is to consider the *strong* Nash equilibria of the game (Auman 1959). A strong Nash equilibria requires that the equilibrium is robust to deviations of sets, or coalitions, of players (voters) rather than the deviations of individual actors. I refer to a measurable set $C \subset N$ as a voter coalition.

Definition 1 *A strategy profile, $s \in S$, is a strong Nash equilibrium if for no voter coalition $C \subset N$ is there a $\hat{s}_C \in A_C$ such that $u_i(\hat{s}_C, s_{\sim C}) > u_i(s)$, $\forall i \in C$.*

¹⁴Similar results obtain if a finite number of voters is assumed but the equilibrium conditions are slightly more complicated, as each vote can have non-negligible effect on the policy outcome, without adding much insight in substantive terms. See, De Sinopoli & Iannantuoni (2007) for further details.

Restricting the analysis to strong Nash equilibria is arguably a strong requirement on the collective rationality of the voters – especially when the number of voters is large – because it relies on coordination among voters. A more satisfying approach would be to model the coordination process but that is beyond the scope of the present paper. However, considering the strong Nash equilibria provides insights into the incentives facing voters that is a useful indication of what might occur if voters could coordinate effectively – whether during a single electoral campaign or over the period of several elections (Fey 1997).

It will be useful to distinguish between two types of equilibria. First, a *coalition equilibrium* refers to an equilibrium in which no party wins a majority of the vote and the policy outcome equals the weighted average of each party's vote shares. Second, a *majoritarian equilibrium* refers to an equilibrium in which some party receives more than a majority of the vote and implements its policy platform.

I begin by considering the possibility of a coalition equilibrium. When no party wins a majority, voters have an incentive to vote for extreme parties because doing so “pulls” the policy outcome towards their ideal point. In a coalition equilibrium there exists a cut-off policy that separates voters voting for ‘left’ and ‘right’ parties. The voters have an incentive to vote for the most extreme parties on the left and the right but a subset of the voters will opt to vote for the second most extreme party (on the left or the right) in order to prevent an extremist party from winning a majority of the vote. The cut-off policy is defined as the policy, p^* , that solves the following equality:

$$p^* = \begin{cases} .5p^l + [F(p^*) - .5]p^r + [1 - F(p^*)]p^l & \text{if } x^m \leq \frac{p^l + p^r}{2} \\ F(p^*)p^l + [.5 - F(p^*)]p^{r-1} + .5p^r & \text{if } x^m > \frac{p^l + p^r}{2}. \end{cases} \quad (3)$$

As $F(x)$ is continuous, the cut-off policy is well-defined. If a strong Nash coalition equilibrium exists, the cut-off policy p^* is also the equilibrium policy outcome. The cut-off policy corresponds to an action profile in which everyone to the left (right) of p^* does as much as they can to pull the policy to the left (right), without creating a majority winner. The action profile may, however, be susceptible to deviations that result in a majority winner. Proposition 2 specifies the conditions for a strong Nash coalition equilibrium. Without loss of generality, I focus on the case when $x^m \leq \frac{p^l + p^r}{2}$. If $x^m > \frac{p^l + p^r}{2}$ the conditions, and the proofs, are symmetric.

Proposition 2 *Suppose $x^m \leq \frac{p^1+p^r}{2}$. A strong Nash coalition equilibria exist if and only if $|x^m - p^{k^m}| \geq |x^m - p^*|$ and $\underline{p}^* \geq \frac{p^1+p^*}{2}$.*

The proof of the proposition, as well as proofs of subsequent propositions, are in the appendix. The existence of a strong Nash coalition equilibrium depends on two conditions. First, the median preferred party can not be located too close to the median party, that is, the median voter must be better off than if his most preferred party were elected. Second, a sufficient number of voters must prefer the coalition policy outcome to the policy platforms of the most extreme party on the left.¹⁵ Whether the condition is satisfied depends on several factors. As Party 1 becomes more extreme and the closer the median voter is to p^* , the more likely the condition is to be satisfied. Note, that p^* is a function of p^1 , p^2 , and p^r . The cutoff policy, p^* , is decreasing in p^1 and p^2 but increasing in p^r . The conditions for existence of a strong Nash coalition equilibrium are somewhat restrictive. For example, the condition $\underline{p}^* \geq \frac{p^1+p^*}{2}$ implies that the distance between the median voter's preferred policy and the most extreme left party must be four times greater than the distance between the median voter's preferred policy and the coalition policy outcome.

Figure 1 demonstrates what a coalition equilibrium might look like in a four party system with an uniform distribution of voters. Three parties receive votes in equilibrium. The voters in the interval $[p^1, \underline{p}^*)$ cast their votes for Party 1, i.e., Party 1 receives just short of a majority of the vote and any positive measure of voters would tip the scale in the party's favor. The voters would like to move the policy further to the left but since they are voting for the most extreme party on the left, their actions must be optimal. Party 2 receives the votes of the voters in the interval $[\underline{p}^*, x^m)$. These voters would also like to move the policy further to the left but any additional mass of votes for Party 1 would turn Party 1 into a majority winner, which would move the policy too far to the left for the voters' taste. The voters in $[x^m, p^*)$ cast their votes for Party 1. The voters in $[x^m, p^*)$ would prefer a policy left of the equilibrium policy outcome. A subset of these voters, with the support of the voters in $[p^1, \underline{p}^*)$, could turn Party 2 into a majority winner. The conditions of the propositions rules this possibility out – the conditions implies that the voter in $[x^m, p^*)$ that is least satisfied with the equilibrium outcome still prefers it to having Party 2's platform implemented. The remaining voters cast their votes for Party 4. Their actions are optimal because they would like to move the policy further to the right.

¹⁵The discussion, like the proposition, assumes that $x^m < \frac{p^1+p^r}{2}$.

Figure 1 also highlights an interesting characteristic of the equilibrium. In the equilibrium shown in the figure, the ideological preferences of the voters are not perfect predictor of which party they support. Generally one would expect the voter's ideological position to have a monotone effect on his vote choice; a voter should vote for a more left-leaning party as he becomes more leftist himself. This is true to some extent in the equilibrium shown in the figure; extremist voters generally vote for extremist parties. Ideology is, however, not a good predictor for centrist voters (who all prefer a move to the left here) as the more left-leaning centrist vote for the less extreme party on the left. Note that this is not a general characteristic of the equilibria and other equilibria may exist in which the voters' choices vary monotonically with their preferences. However, the strategy profile picture in the figure is the one that is most robust to changes in the parameters of the model, i.e., if an equilibrium exists, a voting profile analogous to the one depicted will always exist and it may be the unique equilibrium strategy profile. The non-monotonic effect of ideal policy on vote choice suggests that the empirical specification of the spatial model is not quite as straightforward as commonly is assumed. This is especially relevant to the literature that has sought to pit the spatial model against directional models of voting. Although the model considered here assumes that voters vote on the basis of the proximity of the expected policy outcomes the model predicts voting patterns that would more clearly be associated with directional models of voting.¹⁶

In a majoritarian equilibrium the median voter's most preferred party wins a majority of the vote. Only the median preferred party can win as, by definition, the median preferred party is majority preferred to any other party. The votes cast for any other party have no influence on the policy outcome in any action profile where a single party wins a majority of the vote. Thus, numerous action profiles may be associated with the equilibrium policy outcome. However, it is not the case that any profile such that the median preferred party wins is an equilibrium strategy profile as the whole strategy profile defines the possible payoffs from deviations. In other words, the strategies of voters that don't vote for the median preferred party influence whether voting for the median preferred party is optimal for the remaining voters.

Without loss of generality, assume throughout that $p^{k^m} \geq x^m$. Let $p(C, s)_{min}$ be the

¹⁶Admittedly it is easy to imagine more 'reasonable' factors that might lead voters to violate the simple proximity calculations. See, e.g., (Blais et al. 2006). The important implication, however, is that if one wants to evaluate the validity of different behavioral assumptions it is crucial to understand how institutional structures affect the voters' observed actions.

leftmost policy such that $v_k(\hat{s}_C, s_{\sim C}) < \frac{1}{2}$, $\forall k \in K$ that the coalition $C \subset N$ can achieve by altering its voting strategy. When considering an equilibrium policy outcome, such as p^{k^m} , $p(C, s)_{min}$ indicates whether any (sub)coalition of the voters voting for k^m will find it beneficial to deviate. If the voters voting for k^m are on the left of the political spectrum a deviation can only be beneficial if the deviating coalition can shift the policy to the left of k^m . Since k^m receives a majority of the vote this is always possible as all the voters voting for k^m could switch their votes to some party $k < k^m$. Such deviation would, however, always be blocked by the voters located close to the median voter. The definition of $p(C, s)_{min}$ therefore excludes deviations that result in a majority winner. The policy outcome $p(C, s)_{min}$ is achieved if the coalition votes for the leftmost parties (without any party receiving a majority of the vote). If members are added to the coalition, so that $C' \supset C$, then the leftmost policy attainable will be further to the left, i.e., $p(C', s)_{min} < p(C, s)_{min}$. For simplicity I write $p(C)_{min}$ whenever the dependence of $p(C, s)_{min}$ on s is obvious. The rightmost policy that the coalition C can attain, $p(C, s)_{max}$, is defined in an analogous manner.

Suppose $p^{k^m} > x^m$ and that $k^m \neq r$. Consider a strategy profile such that $s_i = k^m$, $\forall i \in L(p^{k^m})$, which implies that $p(s) = k^m$. By definition all the voters in $L(p^{k^m})$ prefer a policy to the left of p^{k^m} . Hence, voters in $L(p^{k^m})$ will only find it profitable to deviate from k^m if it results in a policy left of the median preferred party's policy. The greatest leftward shift in policy, without creating a new majority winner, is achieved if exactly one-half of the electorate vote for Party 1 while the remainder of the voters in $L(p^{k^m})$ cast their votes for Party 2. Whether such deviation is enough to shift move the policy to the left of p^{k^m} depends on the strategy pursued by the voters in $R(p^{k^m})$. Consider two action profiles, s and s' , such that $s_i = s'_i = k^m$, $\forall i \in L(p^{k^m})$ and $s'_i \geq s_i$, $\forall i \in R(p^{k^m})$, where the inequality is strict for a measurable subset of the voters in $R(p^{k^m})$. As s' places greater weight on parties that are further to the right this implies that $p(L(p^{k^m}), s')_{min} > p(L(p^{k^m}), s)_{min}$. This further implies that $s'_i = r$, $\forall i \in R(p^{k^m})$ maximizes $p(L(p^{k^m}))_{min}$. The existence of a strong Nash majoritarian equilibrium depends on $p(L(p^{k^m}))_{min}$ as Proposition 3 shows.

Proposition 3 *Suppose $p^{k^m} \geq x^m$ and $k^m \neq r$. Then a strong Nash majoritarian equilibrium such that $v_{k^m}(s^*) > \frac{1}{2}$ exists if and only if $p(L(p^{k^m}))_{min} \geq p^{k^m}$.*

Proposition 3 establishes the condition for existence of a strong Nash majoritarian equilibrium but a more detailed characterization of the conditions can be obtained by

taking a closer look at $p(L(p^{k^m}))_{min}$. The proposition does not state the equilibrium strategies of the voters explicitly, as there are multiple (policy equivalent) equilibria for some parameters of the model. It is, however, a simple matter to describe a strategy profile that is a strong Nash equilibrium of the game whenever the condition $p(L(p^{k^m}))_{min} \geq p^{k^m}$ is satisfied. This is the strategy profile constructed in the proof of sufficiency in the proposition, i.e., s such that $s_i = k^m, \forall i \in L(p^{k^m})$ and $s_j = r, \forall j \in R(p^{k^m})$. If $p(L(p^{k^m}), s)_{min} \geq p^{k^m}$ then there exist no profitable deviations for any subset of voters in $L(p^{k^m})$ because it would result in policy to the right of p^{k^m} . It is easy to see what the other equilibria of the game would look like. If it is possible to alter the strategies of the voters of some set (possibly the whole set) of the voters in $R(p^{k^m})$ without violating the condition of the proposition then that strategy profile will also be in equilibrium. As an example suppose the $k^m \neq r - 1$ and that $s'_i = k^m, \forall i \in L(p^{k^m})$ and $s'_j = r - 1, \forall j \in R(p^{k^m})$. If $p(L(p^{k^m}), s')_{min} \geq p^{k^m}$, s' is also an equilibrium strategy profile.

Figure 2 demonstrates how the actions of the voters that don't vote for the majority winner determine whether a given action profile is in equilibrium. For simplicity, I assume that the median preferred party's platform is just to the right of the median voter's preferred policy. Suppose first that the voters to the left of p^{x^m} vote for Party k^m while the remaining voter vote for Party 3. Voters who prefer a policy further to the left of p^{x^m} , $L(p^{x^m})$, can achieve a more favorable outcome by shifting their votes to Party 1 as long as sufficiently many of them prefer that policy outcome. If almost all the voters in $L(p^{x^m})$ switch their votes to Party 1 (some must stay put as otherwise Party 1 wins majority) the policy outcome will equal roughly $\frac{p^1+p^3}{2}$.¹⁷ The figure shows that the policy $\frac{p^1+p^3}{2}$ is to the left of p^{x^m} and the deviation is, therefore, preferred by a large share of the voters in $L(p^{x^m})$. It is then a simple matter to find a subset of voters in $L(p^{x^m})$ that prefer to desert the median preferred party. The action profile were the voters divide their votes between parties 2 and 3 is, thus, not an equilibrium. Now suppose instead that the voters that don't vote for the median preferred party cast their vote for Party 4. Now the leftmost policy the voters in the set $L(p^{x^m})$ can achieve is approximately $\frac{p^1+p^4}{2}$, which is to the right of the median preferred party's position. Thus, no beneficial deviations exist for the leftwing voters and voting for the median preferred party is an optimal strategy. Hence, whether a majoritarian equilibrium exists can depend on whether the voters that don't vote for the median preferred party take radical enough positions.

¹⁷That is, assume that the proportion of voters that vote for Party k^m is very small and, thus, has a negligible effect on the policy outcome.

Note that it is not necessary that all the voters in $L(p^{k^m})$ vote for k^m – it is only necessary that a bare majority of the voters in $L(p^{k^m})$ do so. Since such deviations by a ‘small’ subset of $L(p^{k^m})$ influence neither the outcome nor $p(L(p^{k^m}))_{min}$, the resulting action profile will also be an equilibrium strategy profile. One implication of the multiplicity of equilibria is that a number of parties may receive positive voteshares in equilibrium. Because $x^m < p^{k^m}$ this will be true for any of the equilibria that exist under the conditions of Proposition 3.

The existence of a strong Nash majoritarian equilibrium can be characterized in terms of the parties’ policy positions. It has been shown that the most ‘robust’ equilibrium strategy profile (s^*) is the one in which the voters with ideal points to the left of the median preferred party ($L(p^{k^m})$) vote for the median preferred party while the rest votes for the rightmost party. In this scenario the deviation by the voters on the left ($L(p^{k^m})$) that moves the policy the furthest involves voting for the two leftmost party. I refer to this action profile as s' . In particular, exactly half the electorate votes for Party 1 while the rest of $L(p^{k^m})$ votes for Party 2. The policy outcome then equals

$$p(L(p^{k^m}), s^*)_{min} = p(s') = .5p^1 + [F(p^{k^m}) - F(x^m)]p^2 + [1 - F(p^{k^m})]p^r. \quad (4)$$

We then have a corollary of Proposition 3 that shows how existence depends on the parties’ policy platforms and the voter distribution.

Corollary 4 *Suppose $p^{k^m} \geq x^m$ and $k^m \neq r$. A strong Nash majoritarian equilibrium such that $v_{k^m}(s^*) > \frac{1}{2}$ exists if and only if $.5p^1 + [F(p^{k^m}) - F(x^m)]p^2 + [1 - F(p^{k^m})]p^r \geq p^{k^m}$.*

Thus, $p(L(p^{k^m}), s^*)_{min}$ is a function of the policy platforms of the most extremist parties (including the second most extreme party on the left), the proximity of the median preferred party to the median voter ($p^{k^m} - x^m$) and the density of the voter distribution in the interval $[x^m, p^{k^m}]$. $p(L(p^{k^m}), s^*)_{min}$ is decreasing in p^1 , p^2 and $(p^{k^m} - x^m)$ but increasing in p^r . Similarly, the greater the density of voters in the interval $[x^m, p^{k^m}]$ the lower $p(L(p^{k^m}), s^*)_{min}$ will be. Thus, when the voter distribution approaches the uniform distribution, a certain degree of asymmetry in the locations of the policy platforms of the most extreme parties is necessary to ensure equilibrium existence, i.e., the rightmost party must be located further away from the median voter than the leftmost party/parties.

If, on the other hand, the distribution has sufficient density around the median of the distribution, as when the voters' ideal points are, e.g., normally distributed, then the asymmetry in the extremist parties' policy platforms is less important. The condition for equilibrium becomes more difficult to satisfy as the distance between the median voter's ideal point and the median preferred party's platform increases. There are two reasons for why an increase in distance may not make it possible to sustain an equilibrium. First, the right hand side of the inequality increases as the median preferred party moves away from the median voter's preferred position. Second, the set of voters that prefers a shift in policy to the left of p^{k^m} expands, which means that the coalition of voters preferring a policy further to the left can place greater weight on 'extremist' left parties.

4 Simple Coalition Formation

The previous section demonstrated how the introduction of a minimal form of majoritarianism in the policy making process has substantial implication for the types of equilibria that can be sustained. The majoritarian principle that guides most parliaments does, of course, not only apply to single party majorities. Rather, in minority situations, majority coalitions generally form, which introduces a variety of additional considerations for strategic voters. In this section I extend the model above to a simple coalition formation process that allows insight into how the formation of post-electoral coalitions influences voters' behavior.

The game now consists of two stages. In the first stage, the voters cast their votes, much as in the analysis above. In the second stage, the electoral results are translated into a policy outcome by means of forming a majority coalition.¹⁸ Normally, models of coalition formation assume that some party is designated a formateur. The formateur proposes a coalition and the coalition is formed if it is supported by a majority of the legislature. If not, some other party is designated the formateur. It is common to assume that that the formateur designation rule is either sequential, where the parties' vote shares determine the order in which the parties get to propose a coalition (e.g., Austen-Smith & Banks 1988), or probabilistic, where the likelihood of becoming formateur is proportional

¹⁸The model considered in this section is similar in many ways to the model analyzed by Austen-Smith & Banks (1988) but there are several important differences. The coalition bargaining protocol is different, there is no vote threshold for gaining representation in the legislature, party platforms are exogenously determined, and the focus here is on strong Nash equilibria.

to the party's vote share (e.g., Baron & Diermeier 2001).

Here I assume a particularly simple coalition formation stage. The party that receives the most votes proposes a coalition, which is accepted as long as the coalition has majority support.¹⁹ Thus, there is no status quo policy or a caretaker cabinet in the event the formateur doesn't like any of her options. The policy outcome is the weighted average of the coalition parties' policy platforms. While the coalition formation process clearly abstracts away from real-world coalition formation, it does capture the majoritarian nature of coalition building and allows us to consider the strategic incentives of voters when they must take account of which coalition will form, which is our primary goal here.²⁰

Let $\Omega(K)$ be the set of all subsets of K (the power set of K). Let C_M denote the set of majority coalitions, i.e., $C_M(v) = \{C \in \Omega(K) \mid \sum_C v_k > \frac{1}{2} \text{ and } v_k > 0, \forall k \in C\}$. The leader of the party that receives the most votes is designated the formateur following the election. If two or more parties tie for the first place, the formateur is chosen at random. Party k 's formateur strategy is a mapping $\Gamma_k : v \times p \rightarrow C_M$. The strategy profile for all the parties is Γ . The parties seek to implement a policy as close to their platform as possible. The formateur's optimal strategy is thus simply to pick the coalition that yields the most favorable outcome given the distribution of the vote and the parties' policy platforms. It is a simple matter to verify that the formateur has an incentive to form a coalition with parties that have relatively little support (but are sufficiently large for a majority) and are ideologically similar. Alternatively, she may be able obtain a favorable policy outcome by joining forces with parties on her left and her right. Given a coalition, C , the policy outcome now equals:

$$p^C(s) = \frac{\sum_{k \in C} v_k p^k}{\sum_{k \in C} v_k}. \quad (5)$$

The dependence of p^C on s will be suppressed when it is unambiguous. For a coalition $C = \{k, j\}$, I write p^{jk} whenever party j is the formateur and p^{kj} when k is the formateur.

As in the previous section, the equilibrium policy outcome cannot deviate too far away from the median voter's ideal policy. If that were the case, a majority of the voters would

¹⁹Bäck, Debus & Dumont (2008) examine which factors determine the identity of the formateur and find that the largest party has a substantial advantage.

²⁰It is worth noting that the single-formateur formation process is not unknown as "coalition building" in presidential regimes is essentially coalition formation where only one actor can have the role of the formateur.

benefit by casting a vote for the median preferred candidate. In the coalition equilibria in the previous section we saw how voters have an incentive to vote for extremist parties when the policy outcome is the vote-weighted average of all the parties' policy platforms. Similar incentives are present when coalitions that include only a subset of the parties can form. The voters have an incentive to vote for the coalition members that offer the leftmost and rightmost policy platforms. Furthermore, all voters have an incentive to vote for coalition parties – voting for a non-coalition party generally has no effect on the policy outcome. In equilibrium both these incentives may be tempered by the fact that if a party receives more than half the vote it can form a single-party government. In addition, the voters must bear in mind that changing their voting can result in the transfer of the role of the formateur to a different party and/or the formation of a different coalition.

Consider first the coalition equilibria of the game. In a coalition equilibrium only two parties receive votes. Assume that the median preferred party is to the left of the median voter, i.e., $x^m < \frac{p^{k^m} + p^l}{2}$. Suppose party l is the party on the right that receives votes (and as we shall see, it receives exactly half the vote). Then it must be the case that the left-leaning voters vote for the party on the left that generates the policy outcome most favorable to the median voter – if that were not the case $x^m < \frac{p^{k^m} + p^l}{2}$ implies that the voters in $L(x^m)$ could do better. Formally, for each $l > k^m$, define \bar{k} such that $u_m\left(\frac{p^{\bar{k}} + p^l}{2}\right) \geq u_m\left(\frac{p^k + p^l}{2}\right)$ for all $k \leq k^m$.

Proposition 5 describes the coalition equilibria of the game. Figure 3 offers a sketch of the voters' equilibrium actions. As noted above, only two parties receive votes in equilibrium and they receive an equal number of votes. Subsequently, the two parties form a coalition, and the policy outcome equals the average of the parties' policy platforms. The existence of a coalition equilibrium depends on the proximity of the average policy of the vote-receiving parties to the median voter, i.e., if a majority of voters preferred some party to the coalition policy outcome a beneficial deviation would exist. Thus, any pair of parties that are roughly equidistant from the median voter may satisfy this condition. In figure 3, the policy outcome for the coalition $\{2, 3\}$ is shown but the coalition $\{1, 4\}$ would also satisfy the condition. The conditions for a coalition equilibrium are, however, more easily satisfied for centrist parties than extremist parties.

Proposition 5 *Let s be such that $s_i = \bar{k}$ for all $i \in L(x^m)$ and $s_i = l > k^m$ for all $i \in R(x^m)$. Parties \bar{k} and l receive the same number of votes and Party \bar{k} (or l) is designated formateur. Party \bar{k} forms coalition $\{\bar{k}, l\}$ and the coalition implements $\frac{p^{\bar{k}} + p^l}{2}$. The strategy*

profile (s, Γ) is a strong Nash equilibrium if $u_i \left(\frac{p^{\bar{k}} + p^l}{2} \right) \geq u_i \left(\frac{\frac{1}{2}p^{\bar{k}} + [F(x^j) - F(x^m)]p^r}{\frac{1}{2} + [F(x^j) - F(x^m)]} \right)$ for i such that $x^i = x^j$ for all $x^j \in \left(x^m, \frac{p^{\bar{k}} + p^l}{2} \right)$.

Focusing on the set of voters with ideal points in $\left(x^m, \frac{p^{\bar{k}} + p^l}{2} \right)$ is helpful in generating the intuition behind the equilibrium. Consider the equilibrium in which parties 2 and 3 form a coalition. The voters in $\left(x^m, \frac{p^2 + p^3}{2} \right)$ clearly prefer a policy outcome that is further to the left – yet they cast their votes for Party 3. The reason they do so is that casting a vote for Party 2 would allow Party 2 to form a single-party government and implement policy p^2 . The other option would be to vote for Party 1, in which case coalition $\{1, 2\}$ might form, making these voters even worse off. The voters in $\left(x^m, \frac{p^2 + p^3}{2} \right)$ have another option, i.e., to vote for Party 4 in order to reduce Party 3's influence on the policy outcome. The incentive to do so is countered by the fact that as soon as Party 4 gets any votes, it becomes a viable coalition partner for Party 2. If the set of voters in $\left(x^m, \frac{p^2 + p^3}{2} \right)$ is small this option will be attractive to Party 2 because the policy will be close to p^2 but, at the same time, unattractive to the voters in $\left(x^m, \frac{p^2 + p^3}{2} \right)$. If the set of voters in $\left(x^m, \frac{p^2 + p^3}{2} \right)$ is sufficiently large, a deviation making these voters better off will exist. Note that, the deviating coalition may include some of the voters in $L(x^m)$. This might seem to suggest that a profitable deviation will also exist but this is not the case. If too many voters in $L(x^m)$ deviate, Party 3 becomes the formateur. These considerations give rise to the condition of Proposition 5.

The condition for a coalition equilibrium depend on the location of the median preferred voter and the extent to which parties are located symmetrically about the median voter. That is, some pair of parties must be located sufficiently symmetrically about the median to make the coalition policy outcome more attractive than the median preferred party. The equilibria share some of the characteristics of the coalition equilibrium in the previous section but there are important differences. In equilibrium only two parties receive positive vote shares but the difference is that the tendency to vote for extremist parties is tempered by the fact that a coalition forms following the election. In the present model, the equilibrium coalition can consist of centrist parties.

The proof of proposition 5 shows that surplus coalitions will not form in equilibrium. There are two reasons such coalitions are unstable. First, given a coalition, voters generally have an incentive to vote for the most extreme parties within the coalition. Therefore, some parties must tie in equilibrium. Second, the voters' actions also determine the

identity of the formateur. Facing a lottery over formateurs, the concavity of the voters' utility functions implies that ties are not possible in equilibrium. Some set of voters' will be better off guaranteeing a party a plurality.²¹

Majoritarian equilibria may also exist but the equilibrium conditions are restrictive. The reason is simple. In most action profiles that produce a majority winner it is possible for a small coalition to deviate and achieve a preferable outcome. Assume as before that $p^{k^m} < x^m$. Consider a scenario where $p^{k^m} \neq 1$, $r = 3$, and an action profile where $v_{k^m} > \frac{1}{2}$. If the support for k^m derives from the left-leaning voters the voters with $x^i < p^{k^m}$ can easily engineer a deviation such that a set of measure α (as long as α is not too large) divides its votes between Party 1 and the parties on the right. The reason all of the deviating voters do not cast their votes for Party 1 is that they must ensure that Party k^m prefers forming a coalition with Party 1 and, also, that the role of the formateur is not transferred to a party further on the right. A similar argument applies if k^m 's support comes primarily from the right-leaning voters.

Now consider a scenario in which all the voters vote for the median preferred party as shown in the upper panel of figure A. Then a majority of the voters must change their strategy in order to influence the outcome. Consider a deviation by voters that wish to move the policy to the right. First, note that all the deviating voters cannot vote for Party 3 because then Party 3 can implement its policy platform. Hence, some of the voters must vote for Party 1 but that opens up the possibility that Party 2 forms a coalition with Party 1 (resulting in the policy outcome p^{21}) rather than Party 3 (resulting in p^{23}). Furthermore, if the deviation results in Party 3 becoming the formateur the resulting policy will be roughly half-way between the policy platforms of parties 2 and 3, which is a substantial change in policy from p^2 and greater than some of the members of the deviating coalition can stomach. Hence, a deviation that is beneficial to all the members of the deviating coalition must produce a moderate change in policy. Therefore, the deviating

²¹Even if the technical assumptions driving this argument are modified, e.g., so that a centrist party is chosen in the case of a tie, surplus coalitions would still be unlikely. To gain greater appreciation of why surplus coalitions are unlikely to be equilibrium coalitions consider a case that a priori may seem like a good party for an equilibrium: Suppose parties 1 and 3 are located symmetrically about Party 2, which is located at the median, and voter ideal policies are distributed uniformly on the interval $[p^1, p^3]$. Suppose further that parties 1 and 3 receive the same number of votes but Party 2 is the formateur. The least centrist supporters of Party 2 have a clear incentive to change their vote unless $v_1 = v_2 = v_3$ at which point the identity of the formateur changes if additional voters defect from Party 2. However, given the distribution of preferences, the least centrist supporters of Party 2 will still prefer to change their vote and accept a coalition between parties 1 and 2 (or 2 and 3).

coalition must distribute its votes among other parties and, at the same time, take care that Party 2 finds Party 3 a more desirable coalition partner than Party 1. This may mean that a majority of the deviating coalition actually casts its votes for Party 1. The lower panel of figure A depicts what such deviation might look like.

The derivation of the conditions for the existence of a majoritarian equilibria are somewhat tedious as the above discussion suggests but the conditions are generally rather restrictive. To offer an insight into what the conditions look like I consider the case of three parties where one party is on the left of the median preferred party and one is on the right. Figure 5 illustrates the equilibrium actions.

A couple of remarks are in order before we consider the formal statement of the equilibrium. The voters' actions in the equilibrium depicted in figure 5 are clearly counterintuitive – the voters to the right of the median preferred party vote for the median preferred party while the voters to the left vote for the Party 3. Generally the equilibrium depicted is only one of the equilibria that exist but it is the one most robust to changes in the parameters of the model. Thus, other equilibria, in which the voters' actions correspond better with our intuition about voter behavior, will generally exist. More importantly, the proposition below demonstrates that the conditions for existence of a majoritarian equilibrium are very restrictive – and were we to consider equilibrium refinement's that ruled out the counterintuitive voting behavior shown in figure 5 those conditions would be even more difficult to satisfy.²²

The statement of the proposition requires a couple of definition. Define \hat{x} such that $1 - F(\hat{x}) = \frac{1}{2} - F(p^2)$. Intuitively \hat{x} is defined by the set of the most right leaning voters that must switch their votes to Party 3 for Party 3 to win a majority of the vote, i.e., in Figure 4 this amounts to the number of voters located on the line segment between p^{k^m} and x^m . Define \bar{x} such that $\frac{[1-F(\bar{x})p^1+F(p^2)p^3]}{1-F(\bar{x})+F(p^2)} = \frac{[F(\bar{x})-F(p^2)]p^2+F(p^2)p^3}{F(\bar{x})-F(p^2)+F(p^2)}$, i.e., if the voters to the right of \bar{x} switch their vote from Party 2 to Party 1 then Party 3, the formateur, will be indifferent to which party it forms a coalition with.

Proposition 6 *Let s be such that $s_i = 3$ for all $i \in L(p^2)$ and $s_i = 2$ for all $i \in$*

²²I do not explore further equilibrium refinements. One approach would be restrict attention to strategy profiles in which ideal policy has a monotonic effect on vote choice but the disadvantage of doing so is that it would prevent us from considering whether the spatial model can give rise to behavior that a priori appears better explained by the directional model. Another approach would be to consider something akin to the trembling hand equilibria of the game.

$R(p^2)$. Party 2 receives the majority of the vote and implements its ideal policy. The strategy profile (s, Γ) is a strong Nash equilibrium if i) $\hat{x} < \frac{p^2 + p^2 + p^3}{2}$ and ii) if $p^2 - \frac{[1-2F(p^2)]p^1 + F(p^2)p^2}{[1-2F(p^2)] + F(p^2)} \leq \frac{p^2 + p^3}{2} - p^2$ then $\bar{x} < \frac{p^2 + \frac{[1-F(\bar{x})]p^2 + F(p^2)p^3}{1-F(\bar{x}) + F(p^2)}}{2}$.

The conditions of the proposition are not particularly intuitive but the two conditions are related. Both conditions are satisfied if the density of the voter distribution is sufficiently concentrated around the median voter. The reason is that when a deviation results in a change in the identity of the formateur large policy changes are likely to occur. If the mass of voters is concentrated around the median voter such deviation will require the participation of moderate voters who are less likely to benefit from large deviations.

In sum, the prospects of a majoritarian equilibria are limited. Proposition 6 only characterizes the conditions for a three party contest, leaving open the question whether majoritarian equilibria are more likely when there are more parties? The previous paragraph offers an intuitive answer to that question. When the number of parties increases, the set of majority coalitions that a formateur can form increases weakly and, consequently, the formateur can achieve a more favorable outcome (or at the very least, no worse). This implies that larger policy shifts result from deviations from the equilibrium strategy but this would merely rule out strategies that distribute the vote among many parties. Action profiles such as the one shown to be in equilibrium above would, however, remain in equilibrium even if there are more parties. However, the location of the ‘new’ parties would matter. In particular, the action profile in figure 5 is no longer an equilibrium if a new party appears to the right of the median party. A more ‘robust’ action profile would require the voters on the left to divide their vote among the parties on the right – in which case each of the right parties become more attractive coalition partners for Party 2 and, therefore, opens up new opportunities for beneficial deviations for the voters. In addition, if the new parties are relatively extreme then there may now exist deviations that result in policies that are relatively close to median voter, which implies that a majoritarian equilibria are less likely to exist. The possibility of a majoritarian equilibrium is further reduced because, although they don’t occur in equilibrium, the possibility of surplus coalitions makes it easier for coalitions of voters to engineer small changes in policy that are acceptable to all its members.

In addition, it is natural to question the plausibility of equilibrium strategies which require voters on one end of the political spectrum to vote for a party on the other end

of the spectrum. Considering the majoritarian equilibria nevertheless offers some useful insights into the incentives voters face. First, if one wants to rule out equilibria on the grounds of ‘unreasonableness’, we are left with the conclusion that the conditions for majoritarian equilibria are even more restrictive than suggested by proposition 6. Second, while the equilibrium actions here are extreme, they reflect an interesting incentive facing voters. That is, voting for the (expected) coalition party that will pull policy in the preferred direction is not the only option the voter has. The voter’s other option is to cast a vote for a party that the voter would prefer to be excluded from the governing coalition. By doing so the voter makes the party receiving his vote a less attractive coalition partner for the formateur and, thus, increases the voter’s chances of his preferred coalition forming.²³

5 Discussion

About 60% of all democratic legislative elections held in the world between 1946 and 2000 used some form of proportional representation (Golder 2005).²⁴ Despite the popularity of proportional representation systems our understanding of the dynamics of electoral competition and voting behavior in such systems remains incomplete. Only recently have scholars turned their attention to examining empirically whether voters consider the impact of their vote on policy outcomes when they cast their vote.²⁵

It can also be argued that theoretical treatment of how voters respond to post-electoral policy processes has lagged behind empirical work. While the theoretical literature has not ignored this issue it could be said to have gotten off on the wrong foot. It frequently assumes that the policy outcome in proportional representation systems is the vote-weighted average of the competing parties’ policy platforms. Empirically, as a description of pol-

²³Note, however, that this incentive is a function of the bargaining protocol and the coalition policy being determined using the vote-weighted average of the coalition parties’ policy positions. The incentive is unlikely to occur in a model in which the parties bargain over policy. While accounting for the bargaining over policy would appear the theoretically right approach, the robustness of Gamson’s Law (Gamson 1961) suggests, perhaps, that the application of the vote-weighted policy assumption within the coalition is not an unreasonable approach.

²⁴This number includes electoral systems classified as multi-tier and mixed by Golder (2005), the great majority of which allocate seats proportionally at some tier or party of the system. Both multi-tier and (non-compensatory) mixed systems tend to bias electoral results away from proportional outcomes. Note, however, that voters face largely the same incentives in these systems as in regular proportional representation systems.

²⁵See, e.g., Kedar (2005), Abramson et al. (2007), and Meffert & Gschwend (2007).

icy making in parliamentary democracies, I argue that this assumption is simply wrong. Parliaments are, by and large, majoritarian institutions. *A priori*, that does not imply that the assumption is bad one. All theories make simplifying assumptions and their face validity is not necessarily a good indicator of its usefulness. The assumption may capture some aspects of the political process in a simple way. However, if that is the case, a more detailed modeling of the political process should not alter the predictions of the theory substantially. The first goal of this paper was to demonstrate that this is not the case.

Although this modification does not rule out equilibria in which voters vote for extremist parties, the existence of such equilibria depends on the parties' locations. For example, the presence of centrist party makes an extremist equilibrium unlikely. Thus, in the majoritarian version of the model, the policy outcome will generally be closer to the median voter and generally more than two parties receive votes. While the introduction of a slight majoritarian element does not produce realistic predictions it, nevertheless, demonstrates clearly that the results depend quite heavily on the assumption of vote-weighted average policy outcome. Even the slightest modification of the assumption can lead to different patterns of voting, an increase in the number of vote receiving parties, and different policy outcomes.

The second goal of the paper is to offer insights into how post-electoral coalition formation shapes voters' incentives for strategic behavior. Assuming a simple process of coalition formation I show that voters may face two main types of incentives. First, as policy outcomes are the vote-weighted average of the coalition parties' platforms, voters generally have an incentive to vote for a coalition party in order to edge the policy outcome closer to their preferred policy. A vote for a prospective opposition party is a wasted vote – in the sense that it would have had a direct impact on policy if cast for one of the coalition parties. This incentives reflects two types of considerations that have appeared in the empirical literature. The first addresses findings that voters engage in policy balancing, i.e., attempts to edge policy closer to their most preferred outcome. The second addresses the fact that voters appear not to carry the logic to its logical conclusion, i.e., to vote for the most extreme parties in the legislature, but rather that voters' willingness to depart from voting for their most preferred party is tempered by considerations about which parties are likely to be included in the governing coalition

The second type of strategic incentives concerns the voters' ability to influence policy outcomes by influencing the identity of the formateur as well as how attractive as coalition

partners the other parties are to the formateur.²⁶ While the possibility of influencing who becomes the formateur has straightforward implications for the voters' decisions, the latter incentive results in somewhat counterintuitive actions by the voters. Generally, a non-formateur party becomes less attractive as a coalition partner the more votes it receives. Thus, in order to obtain a favorable policy outcome a voter may have an incentive to vote for a party that they do not want in the governing coalition because it provides the formateur with an incentive to form the voter's preferred coalition. In this sense, a vote for an opposition party is not necessarily a wasted vote. Indeed, the impact on the policy outcome, for sets of voters that are pivotal in this respect, will generally be substantially larger than if they cast their votes for one of the coalition parties. While such consideration would seem to require a lot of voters, Meffert et al. (2008) find evidence that voters that supported the SPD/CDU grand coalition nevertheless voted for the left party for this reason in the 2005 German legislative election.

It is not surprising that the results obtained in the coalition formation game provide a sharper contrast with pure model of vote-weighted policy outcomes. The voters' incentives to vote for extreme parties are tempered by the ability of the parties to form majority coalitions that monopolize policy making. While coalitions of extremist parties are not ruled out, coalitions of moderate parties are more robust to changes in the model's parameters. Again, in comparison with vote-weighted model, policy outcomes tend to be more centrist and more in line with the conventional wisdom about proportional representation systems. Majoritarian equilibria can also occur in the coalition formation game but any action profile that results in a majority winner is highly vulnerable to voter deviations.

The findings in this paper suggest that the assumption of vote-weighted policy outcomes is far from innocuous. This raises serious question about the appropriateness of the assumption when modeling proportional representation systems. Relatively minor modification of this assumption leads to substantively different outcomes – irrespective of whether the modification involves introducing a minimal element of majoritarianism or a simple coalition formation procedure. While the model considered here doesn't allow policy platforms to be determined endogenously, as some of the literature does (e.g., Ortuño-Ortín 1997), it appears clear that the parties' position taking incentives are altered in fundamental ways that would reduce the incentive to adopt extremist positions. Given

²⁶Note that although voters don't face these incentives in equilibrium they are a part of the voters' equilibrium strategies. However, such incentives play an important role if some proportion of the electorate is 'naive' about the policy-making process or votes sincerely.

these findings, it appears particularly ill-advised to take the prediction derived assuming vote-weighted policy outcomes to data. Models of proportional representation should take greater account of the policy making process and, in particular, the fact that coalitions, which monopolize policy making powers, form after elections take place.

A Appendix

Proposition 2 *Suppose $x^m \leq \frac{p^1 + p^r}{2}$. A strong Nash coalition equilibria exist if and only if $|x^m - p^{k^m}| \geq |x^m - p^*|$ and $\underline{p}^* \geq \frac{p^1 + p^*}{2}$.*

Proof: (Sufficiency) $p^* > x^m$ by definition of p^* . Let $\underline{p}^* = x^m - (p^* - x^m)$. Consider a strategy profile s such that $s_i = 1, \forall i \in L(\underline{p}^*) \cup (R(x^m) \cap L(p^*))$, $s_i = 2, \forall i \in R(\underline{p}^*) \cap L(x^m)$, and $s_i = r, \forall i \in R(p^*)$. Then $p(s) = p^*$. Any deviation by C such that $C \cap L(p^*) = \emptyset$ and $C \cap R(p^*) = \emptyset$ can not benefit all $i \in C$. For any deviation s'_C such that $C \subset R(p^*)$, $p(s'_C, s_{\sim C}) < p^*$ and $u_i(s) < u_i(s'_C, s_{\sim C})$ for $i \in C$. Now consider the possible deviations for $C \subset L(p^*)$. For any s'_C such that $(s'_C, s_{\sim C})$ doesn't produce a majority winner we have $p(s'_C, s_{\sim C}) > p^*$ and $u_i(s'_C, s_{\sim C}) < u_i(s)$ for $i \in C$. Thus, any profitable deviation must result in a majority winner. No candidate offers a platform in the interval $[\underline{p}^*, p^*]$ by $|x^m - p^{k^m}| \geq |x^m - p^*|$. Consider a candidate $k \notin \{1, 2\}$ such that $p^k < \underline{p}^*$. Then $v_k(s) = 0$. If k is to win there must exist s'_C such that $v_k(s'_C, s_{\sim C}) > \frac{1}{2}$ and $u_i(s'_C, s_{\sim C}) \geq u_i(s), \forall i \in C$. Then $C \cap [\underline{p}^*, p^*] \neq \emptyset$ but $u_i(s) > u_i(s'_C, s_{\sim C}), \forall i$ such that $x^i \in [\underline{p}^*, p^*]$. Thus, s'_C is not a profitable deviation for all $i \in C$. Now consider candidate 2 and suppose there exists s'_C such that $v_2(s'_C, s_{\sim C}) > \frac{1}{2}$ and $u_i(s'_C, s_{\sim C}) \geq u_i(s), \forall i \in C$. Note that $p^k < \underline{p}^*$. As $v_2(s) = .5 - F(\underline{p}^*)$, the coalition C only needs $F(\underline{p}^*) + \epsilon$ voters to switch their votes from $s_i = 1$ to $s'_i = 2$. All the voters in $L(\underline{p}^*)$ might prefer p^2 to p^* . However, if $C = L(\underline{p}^*)$ and $s'_C = 2$ then $v_2(s'_C, s_{\sim C}) = \frac{1}{2}$ and a coalition outcome with $p(s'_C, s_{\sim C}) > p^*$ obtains. Hence, it must be that $C \cap R(x^m) \cap L(p^*) \neq \emptyset$. However, by definition of \underline{p}^* and $p^2 < \underline{p}^*$, $u_i(s'_C, s_{\sim C}) < u_i(s), \forall i \in R(x^m) \cap L(p^*)$ so we have a contradiction. Finally, consider the possibility of a deviation s'_C such that $v_1(s'_C, s_{\sim C}) > \frac{1}{2}$. No such deviation exists since $u_i(s) > u_i(s'_C, s_{\sim C}), \forall i \in R(\underline{p}^*) \cap L(x^m)$ by $\underline{p}^* \geq \frac{p^1 + p^*}{2}$ and all the other voters in $L(p^*)$ already vote for candidate 1.

(Necessity) First, note that any strong Nash coalition equilibrium will be characterized by extreme voting consistent with the definition of p^* . That is, candidate 1 will receive exactly half the vote, candidate 2 will receive $F(p^*) - .5$, and candidate r the remainder.

To see why that is the case, consider a strategy profile, \hat{s} such that $\hat{s}_i \notin \{1, 2\}$ for some voter (or set of voters) $i \in L(p(\hat{s}))$. Then \hat{s}_i cannot be optimal as the voter perceives himself to influence the outcome of the election, i.e., by voting for candidate 1 or 2 the voter assumes he will edge the policy a little to the left. A similar argument applies to any voter with an ideal point to the right of $p(\hat{s})$ who doesn't vote for candidate r . Thus, only candidates 1, 2, and r can receive a positive vote share in a strong Nash coalition equilibrium. Now, suppose that $|x^m - p^{k^m}| < |x^m - p^*|$. This immediately implies that there exists a profitable deviation for either $L(x^m + \epsilon)$ or $R(x^m - \epsilon)$ that results in the median preferred candidate winning a majority of the vote. To show the necessity of $\underline{p}^* \geq \frac{p^1 + p^*}{2}$, we must construct a strategy profile that minimizes the opportunity for profitable deviations, that is, the strategy profile that will be a strong Nash coalition equilibrium for the largest set of different parameters of the model. Let $L^1 = \{i \in L(p^*) | s_i = 1\}$ and $L^2 = \{i \in L(p^*) | s_i = 2\}$. As $v_1(s) = .5$, deviation by any measurable subset of L^2 is sufficient to make candidate 1 the majority winner. Thus, as the more left-leaning voters are more likely to prefer candidate 1, the lower bound of L^2 determines whether the strategy profile is susceptible to a profitable deviation by some subset of L^2 . Let L_ϵ^2 be a subset of L^2 measuring ϵ such that $x^i < x^j, \forall i \in L_\epsilon^2$ and $\forall j \in L^2 \setminus L_\epsilon^2$. It follows that a strategy profile such that $L^2 = R(x^m) \cap L(p^*)$ provides the greatest guarantee against such deviation. As $v_2(s) = F(p^*) - .5$ a greater subset of L^1 , subset of measure $F(\underline{p}^* + \epsilon)$ to be exact, must vote for candidate 2 to make her the majority winner. Let \bar{L}^1 be the subset of L^1 with the lowest upper bound such that the measure of \bar{L}^1 equals $F(\underline{p}^* + \epsilon)$. The strategy profile that provides the greatest guarantee against a deviation that makes candidate 2 a majority winner maximizes the upper bound of \bar{L}^1 . Note that the upper bound of \bar{L}^1 can never equal p^* as $\bar{L}^1 \subset L^1 \subset L(p^*)$. Let \bar{L}_ϵ^1 be subset of \bar{L}^1 measuring ϵ such that $x^i > x^j, \forall i \in \bar{L}_\epsilon^1$ and $\forall j \in \bar{L}^1 \setminus \bar{L}_\epsilon^1$. The sets \bar{L}_ϵ^1 and L_ϵ^2 can be thought as the location of the 'pivotal voters' with respect to the two types of deviations. Constructing the strategy profile that is the least susceptible to any deviation involves a trade-off as we would like the 'pivotal voters' to be close to p^* . Suppose there is an interval $[\hat{x}, \hat{x} + \epsilon]$ that must contain either \bar{L}_ϵ^1 or L_ϵ^2 . The action profile in which the voters with ideal points in the interval $[\hat{x}, \hat{x} + \epsilon]$ belong to L_ϵ^2 will be less susceptible to a profitable deviation because a deviation by L_ϵ^2 result in the policy p^1 whereas a deviation by \bar{L}^1 would result in p^2 . A voter i such that $x^i \in [\hat{x}, \hat{x} + \epsilon]$ may prefer p^2 to p^* while preferring p^* to p^1 . Thus, the strategy profile least susceptible to profitable deviations will be such that $x^i < x^j, \forall i \in L_\epsilon^2$ and $\forall j \in \bar{L}_\epsilon^1$. The construction of the strategy profile follows from

this observation in a straightforward manner. First, L^2 must be picked as to maximize the ideal points of the voters in L_ϵ^2 subject to the aforementioned constraint. It follows that $s_i = 1, \forall i \in L(\underline{p}^*)$. Note that the measure of the set $L(\underline{p}^*)$ is not sufficiently large for a deviation by the coalition to make candidate 2 a majority winner. The coalition of voters in the interval $[\underline{p}^*, \underline{p}^* + \epsilon]$ corresponds to the set \bar{L}_ϵ^1 and therefore $s_i = 2, \forall i \in [\underline{p}^*, \underline{p}^* + \epsilon]$. As a deviation $s'_i = 1, \forall i \in [\underline{p}^*, \underline{p}^* + \epsilon]$ is sufficient to make candidate 1 the majority winner, it is inconsequential for the possibility of candidate 1 winning a majority which voters belong to $L^2 \setminus L_\epsilon^2$. However, the second objective in constructing the strategy profile was to select the voters in \bar{L}_ϵ^1 so that their ideal points are as far to the right as possible. This is achieved by letting $s_i = 2, \forall i \in R(\underline{p}^*) \cap L(x^m)$, which implies that $s_i = 1, \forall i \in R(x^m) \cap L(\underline{p}^*)$. It is a simple matter to verify that this strategy profile satisfies the definition of p^* . It only remains to check under what condition this strategy profile constitutes a strong Nash coalition equilibrium. First, there exists no s'_C such that $p(s'_C, s_{\sim C}) = p^2$ as, by $|x^m - p^{k^m}| < |x^m - p^*|, u_i(p^*) > u_i(p^2) > u_i(p^1)$ for all $x^i \in [x^m, p^*]$. Second, a deviation s'_C exists if $u_i(p^1) > u_i(p^*)$ for all $i \in L_\epsilon^2$. By construction of s , if $i \in L_\epsilon^2$ then $x^i \in [\underline{p}^*, \underline{p}^* + \epsilon]$. Then $u_i(p^1) > u_i(p^*), \forall i \in L_\epsilon^2$, if $p^* - \underline{p}^* < \underline{p}^* - p^1$. Rearranging the terms we have $\underline{p}^* \geq \frac{p^1 + p^*}{2}$ as in the statement of the proposition. \square

Proposition 3 *Suppose $p^{k^m} \geq x^m$ and $k^m \neq r$. Then a strong Nash majoritarian equilibrium such that $v_{k^m}(s^*) > \frac{1}{2}$ exists if and only if $p(L(p^{k^m}))_{min} \geq p^{k^m}$.*

Proof: (Sufficiency) Suppose $p(L(p^{k^m}))_{min} \geq p^{k^m}$. First, deviation by any coalition C such that $C \cap L(p^{k^m}) \neq \emptyset$ and $C \cap R(p^{k^m}) \neq \emptyset$ can not benefit all members of C . There are two cases. i) Suppose that $k^m = 1$. Consider a strategy profile s^* such that $s_i^* = k^m = 1, \forall i \in L(p^{k^1})$ with $s_j^* > 1, \forall j \in R(p^{k^1})$. By $p^{k^m} > x^m, v_{k^1} > \frac{1}{2}$ and $p(s^*) = p^1$. By $p(L(p^{k^m}))_{min} \geq p^{k^m}, s_C^* = k^1$ is optimal for all $C \subseteq L(p^{k^1})$. Since $v_{k^1} > \frac{1}{2}$, any deviation by $C \subseteq R(p^{k^1})$ does not influence the policy outcome. ii) Suppose that $k^m \geq 2$. Consider a strategy profile s^* such that $s_i^* = k^m, \forall i \in L(p^{k^m} + \epsilon)$ and $s_j^* = r, \forall j \in R(p^{k^m} + \epsilon)$. First, $s_i^* = r$ is optimal for all $C \subset R(p^{k^m} + \epsilon)$ as $v_{k^m}(s_C, s_{\sim C}^*) > \frac{1}{2}$, for all $s_C \in A_C$ and all $C \subset R(p^{k^m} + \epsilon)$. Note also that $s_i^* = r$ is not a weakly dominated strategy for any $i \in R(p^{k^m} + \epsilon)$. To see why that is the case, suppose that candidates m and $m - 1$ receive $\frac{1-\epsilon}{2}$ votes each. By the definition of the median preferred candidate $|p^m - x^m| < |p^{m-1} - x^m|$ and for small enough ϵ the policy outcome associated with the action profile will be less than x^m . As $x^i > x^m$ for all $i \in R(p^{k^m} + \epsilon)$ voting for candidate r is a best response and r is therefore not dominated. $p(L(p^{k^m}))_{min} \geq p^{k^m}$ implies that

no profitable deviation exists for any $C \subseteq L(p^{k^m})$. As no coalition finds it beneficial to deviate s^* is an equilibrium.

(Necessity) Let s^* be an equilibrium strategy with $v_{k^m}(s^*) \geq \frac{1}{2}$. Consider the two cases considered above. i) If $k^m = 1$ then the condition of the proposition is trivially satisfied as the policy outcome can never lie outside the interval $[p^1, p^r]$. ii) Suppose contrary to the statement of the proposition that $p(L(p^{k^m}))_{min} < p^{k^m}$. As there is a continuum of voters, $p(L(p^{k^m}))_{min} < p^{k^m}$ and $k^m \notin \{1, r\}$ implies that there exists $s_{L(p^{k^m})}$ such that $p(s_{L(p^{k^m})}, s_{\sim L(p^{k^m})})$ takes any value in $[p(L(p^{k^m}))_{min}, p^{k^m}]$. Then there exists, by definition of $p(L(p^{k^m}))_{min}$, a coalition $C \subset p(L(p^{k^m}))_{min}$ and s_C such that $u_i(s_C, s_{\sim C}^*) > u_i(s^*)$ for all $i \in C$. Hence, s^* cannot be an equilibrium strategy. \square

Proposition 5 *Let s be such that $s_i = \bar{k}$ for all $i \in L(x^m)$ and $s_i = l > k^m$ for all $i \in R(x^m)$. Parties \bar{k} and l receive the same number of votes and Party \bar{k} (or l) is designated formateur. Party \bar{k} forms coalition $\{\bar{k}, l\}$ and the coalition implements $\frac{p^{\bar{k}} + p^l}{2}$. The strategy profile $(s, \Gamma_{\bar{k}})$ is a strong Nash equilibrium if $u_i\left(\frac{p^{\bar{k}} + p^l}{2}\right) \geq u_i\left(\frac{\frac{1}{2}p^{\bar{k}} + [F(x^j) - F(x^m)]p^r}{\frac{1}{2} + [F(x^j) - F(x^m)]}\right)$ for i such that $x^i = x^j$ for all $x^j \in \left(x^m, \frac{p^{\bar{k}} + p^l}{2}\right)$.*

Proof: Parties \bar{k} and l receive the same number of votes and Party \bar{k} is designated formateur. Party \bar{k} can only form a majority coalition with Party l , i.e., $C_M(v) = \{\{\bar{k}, l\}\}$. Now consider the voters' actions. First, deviation by any coalition C such that $C \cap L\left(\frac{p^{\bar{k}} + p^l}{2}\right) \neq \emptyset$ and $C \cap R\left(\frac{p^{\bar{k}} + p^l}{2}\right) \neq \emptyset$ cannot benefit all $i \in C$. There exist no profitable deviations for $C \subseteq L(x^m)$. By the definition of \bar{k} , some $i \in C$ will be worse off if $C = L(x^m)$. Any other deviation, s'_C , by a measurable set of voters $C \subset L(x^m)$ implies that $v_l(s'_C, s_{\sim C}) > v_{\bar{k}}(s'_C, s_{\sim C})$ in which case Party l is the formateur. As long as $C \neq L(x^m)$, Party l can still form the coalition $\{\bar{k}, l\}$ in which case $p(s'_C, s_{\sim C}) > p(s)$. This further implies that any profitable deviation s'_C for $C \subseteq L\left(\frac{p^{\bar{k}} + p^l}{2}\right)$ must be such that $v_{\bar{k}}(s'_C, s_{\sim C}) \geq v_{\bar{k}}(s)$, i.e., all $i \in C$ will be worse off if Party \bar{k} does not become the formateur. Thus, when considering deviations by $C \subseteq L\left(\frac{p^{\bar{k}} + p^l}{2}\right)$, we can restrict our attention to deviations by $C \subseteq L\left(\frac{p^{\bar{k}} + p^l}{2}\right) \cap R(x^m)$. If s'_C is such that $v_{\bar{k}}(s'_C, s_{\sim C}) > v_{\bar{k}}(s)$ then Party \bar{k} is a majority winner and implements $p^{\bar{k}}$, which leaves all $i \in C$ worse off by the condition of proposition (to see this substitute x^m for x^j in the condition). The only remaining deviations involve some $i \in C \subseteq L\left(\frac{p^1 + p^2}{2}\right) \cap R(x^m)$ casting their votes for Party h , $h > k^m$ and $h \neq l$, in which case Party \bar{k} forms a coalition with Party h if $p_{\{\bar{k}, h\}} < \frac{p^{\bar{k}} + p^l}{2}$. Thus, if voter i prefers $p(s'_C, s_{\sim C})$ to $\frac{p^{\bar{k}} + p^l}{2}$ and voter j prefers $\frac{p^{\bar{k}} + p^l}{2}$ to

$p(s'_C, s_{\sim C})$ then it must be the case that $x^i < x^j$. It follows that if there exist a deviation s'_C by a nonconvex coalition $C \subset L\left(\frac{p^1+p^2}{2}\right) \cap R(x^m)$ such that $u_i(p(s'_C, s_{\sim C})) > u_i\left(\frac{p^1+p^2}{2}\right)$ for all $i \in C$ then there exist a convex coalition $\hat{C} = L(x^j) \cap R(x^m)$ of the same size such that $u_i(p(s'_C, s_{\sim C})) > u_i\left(\frac{p^1+p^2}{2}\right)$ for all $i \in \hat{C}$. The right-most voters in the set \hat{C} , i.e., voters with ideal points close to x^j , are most likely to oppose the deviation as $p(s'_{\hat{C}}, s_{\sim \hat{C}})$ may be too far to the left. A deviation $s''_{\hat{C}}$ such that $h = r$ implies $p(s''_{\hat{C}}, s_{\sim \hat{C}}) \geq p(s'_{\hat{C}}, s_{\sim \hat{C}})$. Therefore, deviations by \hat{C} are most likely to be profitable if they involve casting votes for the right-most candidate, i.e., $h = r$. But by $u_i\left(\frac{p^{\bar{k}}+p^l}{2}\right) \geq u_i\left(\frac{\frac{1}{2}p^{\bar{k}}+[F(x^j)-F(x^m)]p^r}{\frac{1}{2}+[F(x^j)-F(x^m)]}\right)$ for i such that $x^i = x^j$ for all $x^j \in \left(x^m, \frac{p^{\bar{k}}+p^l}{2}\right)$, no such deviation exists for any coalition \hat{C} . Finally, any deviation by $C \subseteq R\left(\frac{p^{\bar{k}}+p^l}{2}\right)$ can only make all $i \in C$ worse off because Part \bar{k} will still be able to form a coalition with Party l . Thus, such deviations will only reduces Party l 's influence on the policy outcome. \square

The main part of the proof of proposition 6 is presented in three lemmas. Given the strategies detailed in proposition 6 a deviation by $C \subseteq R(p^2)$ can result in three types of outcomes; a majority for Party 3, a coalition between parties 2 and 3 formed by Party 2, and coalition formed by Party 3. Deviations resulting in a majority for Party 1 or a coalition between parties 1 and 2 are clearly not beneficial. I consider these in turn. Consider first the possibility of a beneficial deviation that results in Party 3 becoming the majority winner.

Lemma 1 *Let s be such that $s_i = 3$ for all $i \in L(p^2)$ and $s_i = 2$ for all $i \in R(p^2)$. Party 2 receives the majority of the vote and implements its ideal policy. There exists no $C \subseteq R(p^2)$ and s'_C such that $v_3(s'_C, s_{\sim C}) > \frac{1}{2}$ if $\hat{x} < \frac{p^2+p^3}{2}$.*

Proof: Under s , Party 3 falls $\frac{1}{2} - F(p^2)$ votes short of receiving half the vote. By the definition of \hat{x} , if the voters in $R(\hat{x} - \epsilon)$, with ϵ positive but arbitrarily small, vote for Party 3 instead of Party 2 then Party 3 receives a majority of the vote and forms a single party government. All the voters in $R(\hat{x} - \epsilon)$ strictly prefer p^2 to p^3 if $\hat{x} > \frac{p^2+p^3}{2}$, which contradicts the condition of the lemma. \square

The next lemma considers deviations by $C \subseteq R(p^2)$ that result in Party 2 being appointed the formateur. Note that a deviation that leads to a tiny shift in policy (to the right) will be acceptable to almost all members of $R(x^2)$ while large shifts will be opposed by more voters. The voters in $R(p^2)$ can always obtain the policy $\frac{p^2+p^3}{2}$ by ensuring that the vote is split equally between parties 2 and 3. However, it is may be possible to obtain

a coalition between parties 2 and 3 that results in a smaller shift in policy away from the median preferred candidate's position. This possibility involves voters in $C \subseteq R(p^2)$ shifting their votes to candidate 1, thus reducing the weight of Party 3's platform on the coalition policy outcome. While this makes p^{23} agreeable to a larger share of $R(p^2)$ it opens up the possibility that Party 2 will prefer forming a coalition with Party 1. Whether such beneficial deviation exists, thus, depends on Party 2 preferring forming a coalition with Party 3 (rather than Party 1) and that all the members of the deviating coalition prefer the policy outcome p^{23} .

Lemma 2 *Let s be such that $s_i = 3$ for all $i \in L(p^2)$ and $s_i = 2$ for all $i \in R(p^2)$. Party 2 receives the majority of the vote and implements its ideal policy. There exists no beneficial deviation for $C \subseteq R(p^2)$ and s'_C such that $v_2(s'_C, s_{\sim C}) > v_k(s'_C, s_{\sim C}), k = 1, 3$ if $i) \hat{x} < \frac{p^2 + \frac{p^2 + p^3}{2}}{2}$ and $ii) \text{ if } p^2 - \frac{[1-2F(p^2)]p^1 + F(p^2)p^2}{[1-2F(p^2)] + F(p^2)} \leq \frac{p^2 + p^3}{2} - p^2$ then $\bar{x} < \frac{p^2 + \frac{[1-F(\bar{x})]p^2 + F(p^2)p^3}{1-F(\bar{x}) + F(p^2)}}{2}$.*

Proof: If $\frac{1}{2} - F(p^2)$ voters switch their votes from Party 2 to Party 3 then $s_2 = s_3 = \frac{1}{2}$ and a coalition between parties 2 and 3 must form resulting in policy outcome $\frac{p^2 + p^3}{2}$. A deviation $s'_i = 3$ for $i \in C \subseteq R(p^2)$, where C has a measure $\frac{1}{2} - F(p^2)$ is feasible if $u_i(s'_C, s_{\sim C}) > u_i(s)$ for all $i \in C$. If $x^i > x^j$, $u_i(s'_C, s_{\sim C}) > u_i(s)$ implies $u_j(s'_C, s_{\sim C}) > u_j(s)$. Thus, it is sufficient to check whether the voters at the lower bound of the set C that maximizes the lower bound prefer the deviation. As $\frac{1}{2} - F(p^2)$ voters must defect, a deviation is beneficial to all $i \in C$ if voters with ideal point \hat{x} prefer deviation, i.e., $\hat{x} > \frac{p^2 + \frac{p^2 + p^3}{2}}{2}$, which contradicts condition $i)$. Now consider a deviation in which party 1 receives a positive voteshare. By $v_2(s'_C, s_{\sim C}) > v_3(s'_C, s_{\sim C}), C \subseteq R(p^2)$, and $v_2(s'_C, s_{\sim C}) > v_k(s'_C, s_{\sim C}), k = 1, 3$, Party 1 receives at most $1 - 2F(p^2)$ votes. The policy outcome associated with a coalition between Party 1 and Party 2 is $p^{21} = \frac{[1-2F(p^2)]p^1 + F(p^2)p^2}{1-2F(p^2) + F(p^2)}$ while the policy outcome associated with a coalition between Party 2 and Party 3 is $p^{23} = \frac{p^2 + p^3}{2}$. $\Gamma_2 = \{2, 3\}$ if $p^2 - \frac{[1-2F(p^2)]p^1 + F(p^2)p^2}{[1-2F(p^2)] + F(p^2)} \leq \frac{F(p^2)p^2 + F(p^2)p^3}{F(p^2) + F(p^2)} - p^2 = \frac{p^2 + p^3}{2} - p^2$. If the condition fails then no deviation exists in which Party 2 is the formateur and Party 1 receives a positive votes share. If the condition holds, define \bar{x} such that condition holds with equality, i.e., $p^2 - \frac{[1-F(\bar{x})]p^1 + [F(\bar{x}) - F(p^2)]p^2}{[1-F(\bar{x})] + [F(\bar{x}) - F(p^2)]} = \frac{[1-F(\bar{x})]p^2 + F(p^2)p^3}{1-F(\bar{x}) + F(p^2)} - p^2$ subject to $1 - F(\bar{x}) \in [0, 1 - 2F(p^2)]$. If $s'_i = 1$ for all $i \in C = R(\bar{x})$ then Party 2 is indifferent between p^{21} and p^{23} . The deviation is beneficial for all $i \in C$ if it is beneficial for the voters with ideal point \bar{x} . This is the case if $\bar{x} > \frac{p^2 + \frac{[1-F(\bar{x})]p^2 + F(p^2)p^3}{1-F(\bar{x}) + F(p^2)}}{2}$, contradicting condition $ii)$. \square

Lemma 3 *Let s be such that $s_i = 3$ for all $i \in L(p^2)$ and $s_i = 2$ for all $i \in R(p^2)$. Party 2*

receives the majority of the vote and implements its ideal policy. There exist no beneficial deviation for $C \subseteq R(p^2)$ and s'_C such that $\frac{1}{2} > v_3(s'_C, s_{\sim C}) > v_k(s'_C, s_{\sim C}), k = 1, 2$ if $\hat{x} < \frac{p^2 + \frac{p^2 + p^3}{2}}{2}$.

Proof: By $C \subseteq R(p^2)$, $v_3(s'_C, s_{\sim C}) \geq F(p^2)$. Note that more than $F(x^m) - F(p^2)$ voters in $R(p^2)$ must switch their votes to parties 1 and 3 but no more than $F(x^m) - F(p^2)$ to Party 3 if $\frac{1}{2} > v_3(s'_C, s_{\sim C}) > v_k(s'_C, s_{\sim C}), k = 1, 2$. Consider a deviation s'_C such that $s'_i = 1$ for all i such that $x^i \geq \hat{x}$. Recall, \hat{x} is defined by $\frac{1}{2} - F(p^2) = 1 - F(\hat{x})$. Then $p(s'_C, s_{\sim C}) \geq \frac{p^2 + p^3}{2} = p^{32}(s'_C, s_{\sim C})$. That is, Party 3 will only form a coalition with Party 1 if $p^{31}(s'_C, s_{\sim C}) > p^{32}(s'_C, s_{\sim C})$. It is immediately clear that there exist no deviation \hat{s}_C such that $\max\{p^{31}(\hat{s}_C, s_{\sim C}), p^{32}(\hat{s}_C, s_{\sim C})\} < \frac{p^2 + p^3}{2}$. In particular, $p^{32}(\hat{s}_C, s_{\sim C}) < \frac{p^2 + p^3}{2}$ only if $v_2(\hat{s}_C, s_{\sim C}) > v_3(\hat{s}_C, s_{\sim C})$, which contradicts the statement of the lemma. As voting is anonymous and $u_i(p(s'_C, s_{\sim C})) > u_i(p(s))$ implies $u_j(p(s'_C, s_{\sim C})) > u_j(p(s))$ if $x^j > x^i$, the deviation is beneficial for all $i \in C$ if $\hat{x} > \frac{p^2 + \frac{p^2 + p^3}{2}}{2}$. This contradicts the condition of the lemma. \square

Proposition 6 *Let s be such that $s_i = 3$ for all $i \in L(p^2)$ and $s_i = 2$ for all $i \in R(p^2)$. Party 2 receives the majority of the vote and implements its ideal policy. The strategy profile (s, Γ) is a strong Nash equilibrium if i) $\hat{x} < \frac{p^2 + \frac{p^2 + p^3}{2}}{2}$ and ii) if $p^2 - \frac{[1 - 2F(p^2)]p^1 + F(p^2)p^2}{[1 - 2F(p^2)] + F(p^2)} \leq \frac{p^2 + p^3}{2} - p^2$ then $\bar{x} < \frac{p^2 + \frac{[1 - F(\bar{x})]p^2 + F(p^2)p^3}{1 - F(\bar{x}) + F(p^2)}}{2}$.*

Proof: Deviation by any coalition C such that $C \cap L(p^2) \neq \emptyset$ and $C \cap R(p^2) \neq \emptyset$ cannot benefit all $i \in C$. There exist no profitable deviations for $C \subseteq L(p^2)$ as $v_2(s'_C, s_{\sim C}) > \frac{1}{2}$ for all s'_C . Condition i) implies the condition of Lemma 1 and together Lemmas 1-3 show that no coalition profitable deviation exist for $C \subseteq R(p^2)$ given the conditions of the proposition. \square

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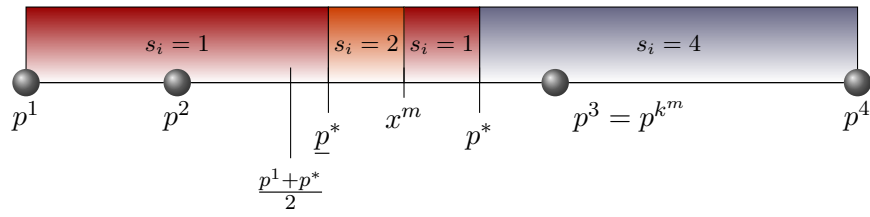


FIGURE 1: COALITION EQUILIBRIA IN A FOUR PARTY SYSTEM (PROPOSITION 2)

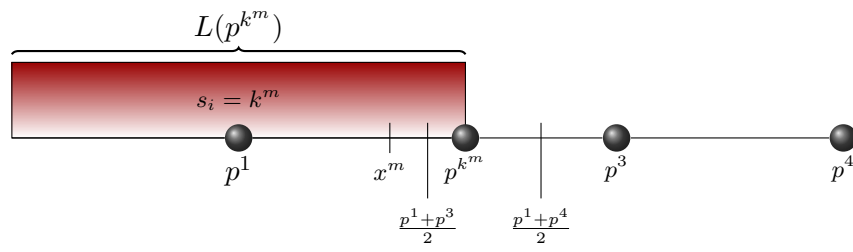


FIGURE 2: MAJORITARIAN EQUILIBRIA (PROPOSITION 3)

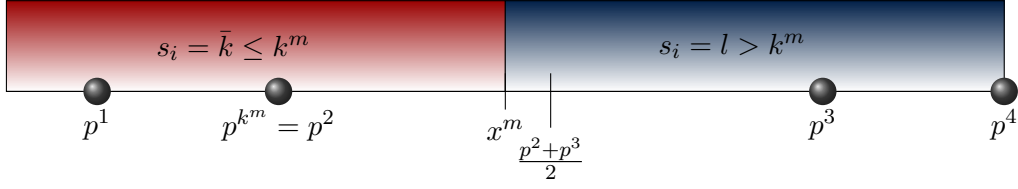


FIGURE 3: COALITION EQUILIBRIA (PROPOSITION 5)

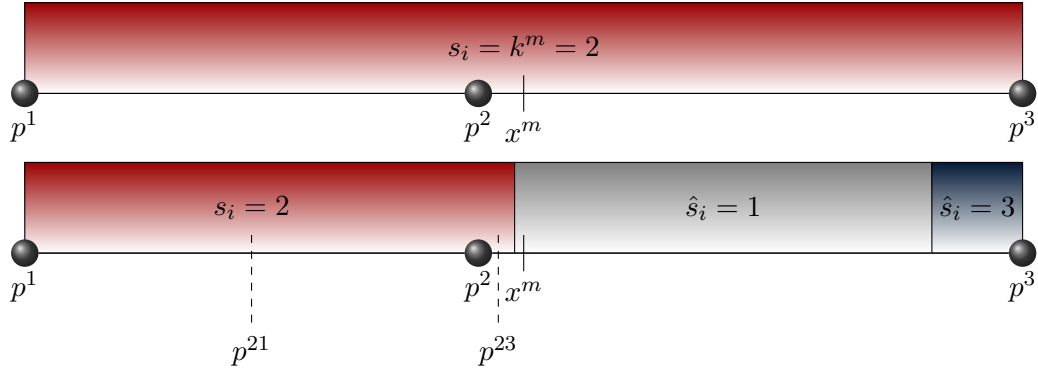


FIGURE 4: DEVIATION FROM AN UNANIMOUS VOTE FOR PARTY 2

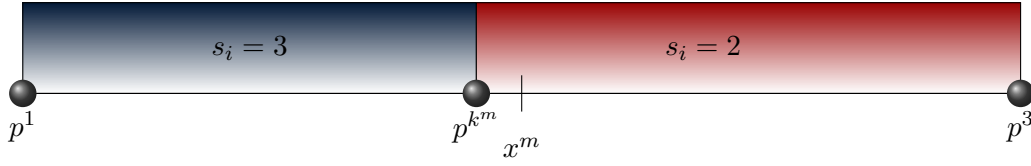


FIGURE 5: MAJORITARIAN EQUILIBRIUM UNDER SIMPLE COALITION FORMATION